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LETTER TO THE EDITOR

Decoherence described by Milburn's theory for the two-photon Jaynes–Cummings model with Stark shift

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Abstract. The Milburn equation without diffusion approximation is solved exactly for the two-photon Jaynes–Cummings model in the presence of Stark shift. This allows the study of the decoherence problem in the whole range of the decoherence parameter γ. The influence of decoherence on atomic inversion is studied. The dynamical behaviour is adjusted in the presence of Stark shift. It is shown that even in the presence of Stark shift the revivals of atomic inversion are destroyed in the decoherence process.

Despite the success of quantum mechanics for over sixty years, many physicists insist on modifying the standard Schrödinger equation to explain why the effects of quantum coherence cannot be observed on the macroscopic scale. One has to introduce the wave-packet-collapse postulate, which is also called the von Neumann projection rule. This rule was accepted as an assumption and added into the calculation by hand. That such a rule remains at the level of an assumption is not satisfactory. This problem is the main concern of quantum measurement theory.

In the past few years alternative quantum theories, in which the state vector represents an individual system and follows a stochastic dynamics, have been introduced [1–7]. The von Neumann projection postulate can be reproduced by taking the ensemble average for the stochastic process [2]. These models not only serve as powerful computational tools [8] but can also be used to generate the data on the statistics governing the quantum jump processed in the same way as in a real experiment [9]. Recently, Milburn has proposed a simple modification of standard quantum mechanics based on the assumption that, with sufficiently short time steps, the system does not evolve continuously under unitary evolution but rather in a stochastic sequence of identical unitary transformations [10]. This assumption leads to a modification of the von Neumann equation which contains a γ term responsible for the decay of quantum coherence in the energy basis while the energy conservation also holds. Previously, Moya-Cessa *et al* [11] constructed a formal solution of the Milburn equation for the one-photon Jaynes–Cummings model (JCM) by using superoperator techniques. This technique has been generalized to use the intensity-dependent JCM [12] and the two-photon JCM [13] to investigate the influence of decoherence on the non-classical properties in these models.

Since the development of new experimental techniques [14] based on the use of Rydberg atoms in high-Q microcavities opened new vistas in the exploration of non-classical effects in the atom–field interaction, much theoretical work has been done on the JCM and its generalizations. The two-photon JCM, which will be of interest to us in this letter, has attracted considerable attention [15–18] because of its relevance to the study of the coupling between a single atom and a single-mode cavity field with the atom making two-photon transitions. When the Stark shift is included, the dynamical behaviour is significantly affected. In a previous letter [13] we obtained the exact solution of the Milburn equation for the two-photon JCM. The particular interests of this letter are to obtain the exact solution of the Milburn equation for the two-photon JCM including the Stark effects both in the case when the diffusion approximation is performed and in the case when no diffusion approximation is made, and to use the solution to investigate the influence of decoherence on the behaviour of atomic inversion. Our method is based on the recent approach to obtaining the solution of the Milburn equation without the diffusion approximation (see below), which also provides an alternative method for solving the Milburn equation [19]. In order to make our presentation self-contained, a brief presentation of this method [19] is provided.

In standard quantum mechanics the change in the state of the quantum system in a time interval $(t, t+\tau)$ is given by the unitary transformation

$$\hat{\rho}(t+\tau) = \exp\left(-\frac{\mathrm{i}\hat{H}\tau}{\hbar}\right)\hat{\rho}(t)\exp\left(-\frac{\mathrm{i}\hat{H}\tau}{\hbar}\right) \tag{1}$$

where $\hat{\rho}(t)$ is the density operator and equation (1) is valid for arbitrarily large or small values of τ. Milburn has replaced the above evolution with some new postulates. For sufficiently short time steps the system does not evolve continuously under the unitary transformation (1) but rather it changes stochastically. If the probability that the state of the system changes is $p(\tau)$, then equation (1) is modified as

$$\hat{\rho}(t+\tau) = \exp\left(-\frac{\mathrm{i}}{\hbar}\theta(\tau)\hat{H}\right)\hat{\rho}(t)\exp\left(\frac{\mathrm{i}}{\hbar}\theta(\tau)\hat{H}\right) \tag{2}$$

where $\theta(\tau)$ is some function of τ. It is also necessary for $p(\tau) \to 1$ and $\theta(\tau) \to \tau$ for values of τ which are sufficiently large; when τ approaches zero, it is postulated that $\lim_{\tau\to 0}\theta(\tau) = \theta_0 = 1/\gamma$. A possible choice of $\theta(\tau)$ is $\theta(\tau) = \tau/p(\tau)$. Dividing the time interval $(0, t)$ into K steps of length τ so that $t = K\tau$, then

$$\begin{aligned}\hat{\rho}(t) &= \lim_{\tau\to 0}\sum_{k=0}^{K}\binom{K}{k}(p(\tau))^k[1-p(\tau)]^{K-k}\hat{\mathcal{R}}^k(\tau)\hat{\rho}(0)\\ &= \lim_{\tau\to 0}[1+p(\tau)(\hat{\mathcal{R}}(\tau)-1)]^{t/\tau}\hat{\rho}(0)\end{aligned} \tag{3}$$

where $\hat{\mathcal{R}}(\tau)\hat{\rho}(0) = \exp(-\mathrm{i}\theta(\tau)\hat{H})\hat{\rho}(0)\exp(\mathrm{i}\theta(\tau)\hat{H})$. Then an equation governing $\hat{\rho}(t)$ is obtained as

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = \gamma\left\{\exp\left[\frac{-\mathrm{i}}{\hbar\gamma}\hat{H}\right]\hat{\rho}(t)\exp\left[\frac{\mathrm{i}}{\hbar\gamma}\hat{H}\right] - \hat{\rho}(t)\right\}. \tag{4}$$

Expanding equation (4) to the first order of $1/\gamma$ we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -\frac{\mathrm{i}}{\hbar}[\hat{H},\hat{\rho}] - \frac{1}{2\hbar^2\gamma}[\hat{H},[\hat{H},\hat{\rho}]]. \tag{5}$$

This is the diffusion approximation equation of (4). In accordance with [11–13] equation (5) is called the Milburn equation and equation (4) is thus called the Milburn equation without the diffusion approximation.

Actually, equation (3) is the formal solution of equation (4). The difficulty in obtaining the explicit solution is that in equation (3) the operator-valued limit is involved which means that the usual method for dealing with c-number valued limits cannot be applied directly. This difficulty can be overcome by properly selecting a basis and computing the action of the superoperator $\hat{R}(\tau)$ on the initial density operator $\hat{\rho}(0)$ to transfer the operator-valued limit to a c-number valued limit.

In this letter our main concern is the resonant two-photon JCM with Stark effect [15] with its Hamiltonian expressed as

$$\hat{H} = \hbar\omega a^+a + \hbar\omega\sigma_z + g(a^2\sigma_+ + a^{+2}\sigma_-) + (\beta_1|e\rangle\langle e| + \beta_2|g\rangle\langle g|)\otimes\hat{n} \tag{6}$$

where a and a^+ are the annihilation and creation operators for the field with frequency ω respectively, g is the atom–field coupling constant, $\sigma_+ = \frac{1}{2}(\sigma_x + \sigma_y)$ and $\sigma_- = \frac{1}{2\mathrm{i}}(\sigma_x - \sigma_y)$ where σ_x, σ_y and σ_z are Pauli matrices for the system in the representation of the ground state $|g\rangle$ and excited state $|e\rangle$ of the two-level atom and β_1 and β_2 denote the parameter of the dynamic Stark shift of the atom. In this letter we take $\hbar = 1$.

Our approach to finding the exact solution of equation (4) is divided into several steps. We should first solve the usual von Neumann equation, i.e. compute the action of $\hat{R}(\tau)$ on the initial density matrix $\hat{\rho}(0)$.

We divide the Hamiltonian into a sum of two terms which commute with each other:

$$\hat{H} = \hat{H}_0 + \hat{H}_I \qquad [\hat{H}_0, \hat{H}_I] = 0 \tag{7}$$

where

$$\hat{H}_0 = \begin{pmatrix} \omega(\hat{n}+1) + r_1(\hat{n}) & 0 \\ 0 & \omega(\hat{n}-1) - r_2(\hat{n}) \end{pmatrix} \tag{8}$$

$$\hat{H}_1 = \begin{pmatrix} \frac{1}{2}(\beta_1 - \beta_2)\hat{n} - \beta_2 & ga^2 \\ ga^{+2} & -\frac{1}{2}(\beta_1 - \beta_2)\hat{n} + \beta_1 \end{pmatrix}. \tag{9}$$

In equations (8) and (9) we have defined

$$r_1(\hat{n}) = \tfrac{1}{2}(\beta_1 + \beta_2)\hat{n} + \beta_2 \qquad r_2(\hat{n}) = -\tfrac{1}{2}(\beta_1 + \beta_2)\hat{n} + \beta_1. \tag{10}$$

Then $\exp(-\mathrm{i}\hat{H}_0\theta(\tau))$ and $\exp(-\mathrm{i}\hat{H}_I\theta(\tau))$ are obtained straightforwardly from

$$\exp(-\mathrm{i}\hat{H}_0\theta(\tau)) = \begin{pmatrix} \mathrm{e}^{-\mathrm{i}\theta(\tau)(\omega(\hat{n}+1)+r_1(\hat{n}))} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\theta(\tau)(\omega(\hat{n}-1)-r_2(\hat{n}))} \end{pmatrix} \tag{11}$$

$$\exp(-\mathrm{i}\hat{H}_I\theta(\tau)) = \begin{pmatrix} \hat{J}_{11} & \hat{J}_{12} \\ \hat{J}_{21} & \hat{J}_{22} \end{pmatrix} \tag{12}$$

where

$$\hat{J}_{11} = \cos(\Omega_+(\hat{n})\theta(\tau)) - \mathrm{i}\left(\frac{\sin(\Omega_+(\hat{n})\theta(\tau))}{\Omega_+(\hat{n})}\right) r_1(\hat{n}) \tag{13}$$

$$\hat{J}_{12} = -\mathrm{i}\left(\frac{\sin(\Omega_+(\hat{n})\theta(\tau))}{\Omega_+(\hat{n})}\right) ga^2 \tag{14}$$

$$\hat{J}_{21} = -\mathrm{i}ga^{+2}\left(\frac{\sin(\Omega_+(\hat{n})\theta(\tau))}{\Omega_+(\hat{n})}\right) \tag{15}$$

$$\hat{J}_{22} = \cos(\Omega_-(\hat{n})\theta(\tau)) - \mathrm{i}\left(\frac{\sin(\Omega_-(\hat{n})\theta(\tau))}{\Omega_-(\hat{n})}\right) r_2(\hat{n}) \tag{16}$$

where we have defined

$$\Omega_+(\hat{n}) = \sqrt{r_1^2(\hat{n}) + g^2(\hat{n}+1)(\hat{n}+2)} \tag{17}$$

$$\Omega_-(\hat{n}) = \sqrt{r_2^2(\hat{n}) + g^2\hat{n}(\hat{n}-1)}. \tag{18}$$

Then, $\exp(-\mathrm{i}\hat{H}\theta(\tau))$ can be obtained by calculating $\exp(-\mathrm{i}\hat{H}_0\theta(\tau))\exp(-\mathrm{i}\hat{H}_I\theta(\tau))$. The action of $\hat{R}(\tau)$ on initial density operator $\hat{\rho}(0)$ can be easily obtained. Here we assume initially that the atom is prepared in its excited state $|e\rangle$. For an arbitrary initial field the initial density operator $\hat{\rho}(0)$ is written as $\hat{\rho}(0) = \sum_{n,m=0}^{\infty} \rho_{n,m}|n\rangle\langle m| \otimes |e\rangle\langle e|$. For the later convenience we express $\hat{R}(\tau)\hat{\rho}(0)$ in the number representation

$$\hat{R}(\tau)\hat{\rho}(0) = \begin{pmatrix} \hat{S}_{11} & \hat{S}_{12} \\ \hat{S}_{21} & \hat{S}_{22} \end{pmatrix} \tag{19}$$

where

$$\begin{aligned}\hat{S}_{11} = \tfrac{1}{4}\sum_{n,m=0}^{\infty} \rho_{n,m}|n\rangle\langle m| \bigg\{ & \left(1 - \frac{r_1(n)}{\Omega_+(n)}\right)\left(1 + \frac{r_1(m)}{\Omega_+(m)}\right) g_+(n,m) + \left(1 - \frac{r_1(n)}{\Omega_+(n)}\right) \\ & \times \left(1 - \frac{r_1(m)}{\Omega_{+m}}\right) g_-(n,m) + \left(1 + \frac{r_1(n)}{\Omega_+(n)}\right)\left(1 - \frac{r_1(m)}{\Omega_+(m)}\right) f_+(n,m) \\ & + \left(1 + \frac{r_1(n)}{\Omega_+(n)}\right)\left(1 + \frac{r_1(m)}{\Omega_+(m)}\right) f_-(n,m) \bigg\}\end{aligned} \tag{20}$$

$$\begin{aligned}\hat{S}_{12} = \tfrac{1}{4}\sum_{n,m=0}^{\infty} \rho_{n,m} & \left(\frac{g\sqrt{(m+1)(m+2)}}{\Omega_+(m)}\right)|n\rangle\langle m+2| \\ & \times \left[\left(1 - \frac{r_1(n)}{\Omega_+(n)}\right)(g_+(n,m) - g_-(n,m)) \right. \\ & \left. - \left(1 + \frac{r_1(n)}{\Omega_+(n)}\right)(f_+(n,m) - f_-(n,m))\right]\end{aligned} \tag{21}$$

$$\begin{aligned}\hat{S}_{21} = -\tfrac{1}{4}\sum_{n,m=0}^{\infty} \rho_{n,m} & \left(\frac{g\sqrt{(n+1)(n+2)}}{\Omega_+(n)}\right)|n+2\rangle\langle m| \\ & \times \left[\left(1 + \frac{r_1(m)}{\Omega_+(m)}\right)(g_+(n,m) - f_-(n,m)) \right. \\ & \left. + \left(1 - \frac{r_1(m)}{\Omega_+(m)}\right)(g_-(n,m) - f_+(n,m))\right]\end{aligned} \tag{22}$$

$$\begin{aligned}\hat{S}_{22} = -\tfrac{1}{4}\sum_{n,m=0}^{\infty} \rho_{n,m} & \left(\frac{g^2\sqrt{(n+1)(n+2)(m+1)(m+2)}}{\Omega_+(n)\Omega_+(m)}\right)|n+2\rangle\langle m+2| \\ & \times [g_+(n,m) - g_-(n,m) + f_+(n,m) - f_-(n,m)]\end{aligned} \tag{23}$$

were we have defined

$$g_\pm(n,m) = \exp[-\mathrm{i}\theta(\tau)(\omega(n-m) + \tfrac{1}{2}(\beta_1+\beta_2)(n-m)) - (\Omega_+(n) \pm \Omega_+(m))] \tag{24}$$

$$f_\pm(n,m) = \exp[-\mathrm{i}\theta(\tau)(\omega(n-m) + \tfrac{1}{2}(\beta_1+\beta_2)(n-m)) + (\Omega_+(n) \pm \Omega_+(m))]. \tag{25}$$

If in equations (19)–(25) the $\theta(\tau)$ is replaced by t, then they compose the solution of the two-photon JCM with the Stark effect governed by the von Neumann equation. $\hat{R}^k(\tau)\hat{\rho}(0)$ can be obtained by replacing $\theta(\tau)$ with $k\theta(\tau)$ in equations (24)–(31). After substituting equation (19) into equation (3) the exponent k now appears only in the c-number part of the expression; with such a situation the limit of equation (3) can be performed. The following formula will be used repeatedly:

$$\lim_{\tau\to 0}\sum_{k=0}^{K}\binom{K}{k}p^k(\tau)(1-p(\tau))^{K-k}\mathrm{e}^{\mathrm{i}\theta(\tau)kq} = \lim_{\tau\to 0}[1+p(\tau)(\mathrm{e}^{\mathrm{i}\theta(\tau)q}-1)]^{t/\tau}$$
$$= \exp[-\gamma t(1-\exp(\mathrm{i}q/\gamma))] \tag{26}$$

where q is an arbitrary k-independent function. In the above calculation the properties of $\theta(\tau)$, such as $\lim_{\tau\to 0}\theta(\tau) = 1/\gamma$, and the relation between $\theta(\tau)$ and $p(\tau)$ have been used. The density operator $\hat{\rho}(t)$ can now be obtained with the help of equation (26):

$$\hat{\rho}(t) = \begin{pmatrix} \hat{S}'_{11} & \hat{S}'_{12} \\ \hat{S}'_{21} & \hat{S}'_{22} \end{pmatrix} \tag{27}$$

where

$$\hat{S}'_{11} = \tfrac{1}{4}\sum_{n,m=0}^{\infty}\rho_{n,m}|n\rangle\langle m|\Bigg\{\left(1-\frac{r_1(n)}{\Omega_+(n)}\right)\left(1+\frac{r_1(m)}{\Omega_+(m)}\right)g'_+(n,m)+\left(1-\frac{r_1(n)}{\Omega_+(n)}\right)$$
$$\times\left(1-\frac{r_1(m)}{\Omega_+(m)}\right)g'_-(n,m)+\left(1+\frac{r_1(n)}{\Omega_+(n)}\right)\left(1-\frac{r_1(m)}{\Omega_+(m)}\right)f'_+(n,m)$$
$$+\left(1+\frac{r_1(n)}{\Omega_+(n)}\right)\left(1+\frac{r_1(m)}{\Omega_+(m)}\right)f'_-(n,m)\Bigg\} \tag{28}$$

$$\hat{S}'_{12} = \tfrac{1}{4}\sum_{n,m=0}^{\infty}\rho_{n,m}\left(\frac{g\sqrt{(m+1)(m+2)}}{\Omega_+(m)}\right)|n\rangle\langle m+2|$$
$$\times\left[\left(1-\frac{r_1(n)}{\Omega_+(n)}\right)(g'_+(n,m)-g'_-(n,m))\right.$$
$$\left.-\left(1+\frac{r_1(n)}{\Omega_+(n)}\right)(f'_+(n,m)-f'_-(n,m))\right] \tag{29}$$

$$\hat{S}'_{21} = -\tfrac{1}{4}\sum_{n,m=0}^{\infty}\rho_{n,m}\left(\frac{g\sqrt{(n+1)(n+2)}}{\Omega_+(n)}\right)|n+2\rangle\langle m|$$
$$\times\left[\left(1+\frac{r_1(m)}{\Omega_+(m)}\right)(g'_+(n,m)-f'_-(n,m))\right.$$
$$\left.+\left(1-\frac{r_1(m)}{\Omega_+(m)}\right)(g'_-(n,m)-f'_+(n,m))\right] \tag{30}$$

$$\hat{S}'_{22} = -\tfrac{1}{4}\sum_{n,m=0}^{\infty}\rho_{n,m}\left(\frac{g^2\sqrt{(n+1)(n+2)(m+1)(m+2)}}{\Omega_+(n)\Omega_+(m)}\right)|n+2\rangle\langle m+2|$$
$$\times[g'_+(n,m)-g'_-(n,m)+f'_+(n,m)-f'_-(n,m)]. \tag{31}$$

In equations (28)–(31) we have defined

$$g'_{\pm}(n,m) = \exp\left\{ -\gamma t\left[1 - \exp\left[-\frac{\mathrm{i}}{\gamma}(\omega(n-m) + \tfrac{1}{2}(\beta_1+\beta_2)(n-m)) - (\Omega_+(n) \pm \Omega_+(m))\right]\right]\right\} \tag{32}$$

$$f'_{\pm}(n,m) = \exp\left\{ -\gamma t\left[1 - \exp\left[-\frac{\mathrm{i}}{\gamma}(\omega(n-m) + \tfrac{1}{2}(\beta_1+\beta_2)(n-m)) + (\Omega_+(n) \pm \Omega_+(m))\right]\right]\right\}. \tag{33}$$

Equations (27)–(33) now compose the solution of equation (4), i.e. the Milburn equation without the diffusion approximation. When $\gamma \gg 1$ the diffusion approximation can be performed, the higher orders of $1/\gamma$ in equations (32) and (33) can be omitted. In this situation $g'_{\pm}(n,m)$ and $f'_{\pm}(n,m)$ can be written as

$$g'_{\pm}(n,m) = \exp\{-\mathrm{i}t[\omega(n-m) + \tfrac{1}{2}(\beta_1+\beta_2)(n-m) - (\Omega_+(n) \pm \Omega_+(m))]\} \times \exp\left\{\frac{t}{2\gamma}[\omega(n-m) + \tfrac{1}{2}(\beta_1+\beta_2)(n-m) - (\Omega_+(n) \pm \Omega_+(m))]^2\right\} \tag{34}$$

$$f'_{\pm}(n,m) = \exp\{-\mathrm{i}t[\omega(n-m) + \tfrac{1}{2}(\beta_1+\beta_2)(n-m) + (\Omega_+(n) \pm \Omega_+(m))]\} \times \exp\left\{\frac{t}{2\gamma}[\omega(n-m) + \tfrac{1}{2}(\beta_1+\beta_2)(n-m) + (\Omega_+(n) \pm \Omega_+(m))]^2\right\}. \tag{35}$$

Then equations (27)–(31), (34) and (35) are the solution of the Milburn equation (5).

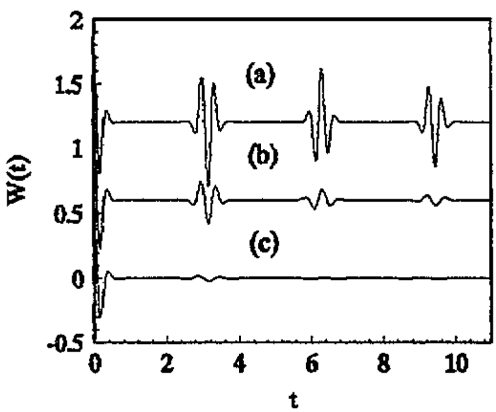

Figure 1. The atomic inversion as a function of time in the absence of the Stark shift for $|\alpha|^2 = 8$ and $g = 1$. In curve (a) $[W(t) + 1.2]$, $\gamma = 3000$, in curve (b) $[W(t)+0.6]$, $\gamma = 400$ and in curve (c) $[W(t)]$, $\gamma = 100$.

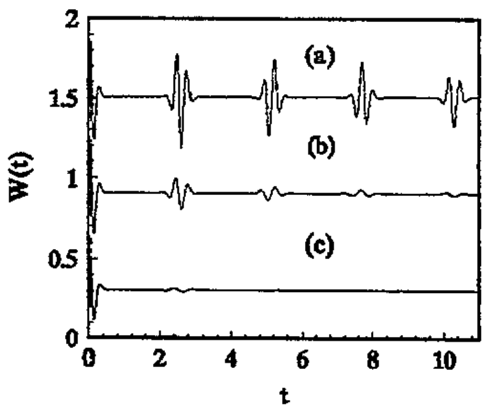

Figure 2. The atomic inversion as a function of time in the presence of the Stark shift with $\beta_1 = 0.6$ and $\beta_2 = 0.8$. The other parameters are the same as those in figure 1.

The collapses and revivals of atomic inversion are the non-classical effects of JCM which originate from the quantum coherences built up during the interaction between the atom and the field [20]. Here we assume the field is initially prepared in the coherent state $|\alpha\rangle$. In this case, $\rho_{n,m} = \exp(-|\alpha|^2)\alpha^n\alpha^{*m}/\sqrt{n!m!}$. By using the solution of equation (4) the atomic inversion is easy to obtain:

$$W(t)=\frac{1}{4}\sum_n |\rho_{nn}|^2\left\{\frac{4g^2(n+1)(n+2)}{\Omega_+^2(n)}\exp\left(-2\gamma t\sin^2\left(\frac{\Omega_+(n)}{\gamma}\right)\right)\cos\left(\gamma t\sin\left(\frac{2\Omega_+(n)}{\gamma}\right)\right)\right.$$
$$\left.+\left[\left(1-\frac{r_1(n)}{\Omega_+(n)}\right)^2+\left(1+\frac{r_1(n)}{\Omega_+(n)}\right)^2-\frac{2g^2(n+1)(n+2)}{\Omega_+^2(n)}\right]\right\}. \qquad (36)$$

When $\gamma \gg 1$ the diffusion approximation can be performed and the atomic inversion is reformulated as

$$W(t)=\frac{1}{4}\sum_n |\rho_{nn}|^2\left\{\frac{4g^2(n+1)(n+2)}{\Omega_+^2(n)}\exp\left(-\frac{2\Omega_+^2(n)t}{\gamma}\right)\cos(2\Omega_+(n)t)\right.$$
$$\left.+\left[\left(1-\frac{r_1(n)}{\Omega_+(n)}\right)^2+\left(1+\frac{r_1(n)}{\Omega_+(n)}\right)^2-\frac{2g^2(n+1)(n+2)}{\Omega_+^2(n)}\right]\right\}. \qquad (37)$$

Equation (37) can also be obtained by using the solution of the Milburn equation (27)–(31), (34) and (35) which is valid for $\gamma \gg 1$. In this case the decaying of atomic inversion becomes faster with the decreasing of γ. When $\beta_1 = \beta_2 = 0$, i.e. when there is no Stark shift, equation (37) reduces to the results in [13]. It is shown that when time approaches infinity the oscillating term in equation (37) will disappear even in the presence of Stark shift and when the range of the parameter $\gamma \gg 1$ does not hold, it is the decay factor $\exp(-2\gamma t\sin^2((\Omega_+(n))/\gamma))$ that is responsible for the destruction of the revivals of atomic inversion. When γ approaches infinity the evolution of the solution of equations (4) and (5) reduces to that governed by von Neumann equation and in this case decoherence has no influence on the behaviour of atomic inversion. Figures 1 and 2 are presented to illustrate the influence of decoherence on the revivals of atomic inversion for different values of the decoherence parameter γ in the cases with and without Stark shift.

In conclusion, we have obtained the solution of the Milburn equation without the diffusion approximation for the two-photon JCM in the presence of Stark shift. When $\gamma \gg 1$ the solution of the Milburn equation can be obtained by expanding the exponent of the solution of equation (4) to the first order of $1/\gamma$. The atomic inversion is obtained by using the solution of equations (4) and (5). It is shown that when the Stark shift is included the behaviour of atomic inversion is adjusted. However, the revivals of atomic inversion will be destroyed when time approaches infinity.

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